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Overcoming Math Anxiety in Philosophy Students: Strategies for a Course on Formal Logic

Introduction

When taking a leisurely stroll through the typical curriculum of a philosophy program one might, as many students do, stumble on a course in formal logic (MacPherson, 2016, p. 123). This course is not like the others; whereas most classes are train the students to read and reason about vast amounts of text, logic is essentially taught as a mathematical subject. This sudden switch in gear often taps into students pre-existing anxieties towards mathematics which, in turn, inhibit learning. As such, courses in logic tend to gain a reputation for being very difficult and an overall unpleasant experience even among the students who pass.

That this problem is familiar to many inhabitants of philosophy departments should come as little surprise considering the prevalence of math anxiety among university students. As Dowker et. al. (2016) claim in a widely cited survey article on the subject "There is no doubt, even when taking the lowest estimates, that it is a very significant problem. (p. 3)".[1] Apart from the studies cited by Dowker et. al., the only other survey I can find on prevalence among university students comes from the University of Granada (2009), which found that six out of ten first-year university students across subjects showed symptoms of significant math anxiety.[2] Further, as is revealed by a meta-analysis performed by Hembree (1990, p. 41), on average students in the humanities exhibit higher math anxiety than students in natural, social, and health sciences. This is well in line with the experience that MacPherson (2016, p. 126) notes of teaching formal logic to philosophy students.

As I will argue below, this presents a serious problem for teaching formal logic. My goal here is to present some strategies that a teacher can adopt to deal with this problem during a course. By that I mean ways which try to *mitigate* the effects of math anxiety on learning outcomes rather than trying to cure it. While it would be wonderful to cure the students' condition, that would seem to ask too much of an introductory course in logic. But before I can get to strategies, I must begin with a brief overview of what math anxiety is and what factors affect it.

[1] Strangely, Dowker et. al. (2016, p. 3) claim that Richardson and Suinn (1972) estimate that 11% of university students show high enough levels of math anxiety to need counseling and that Betz (1978) estimates that 68% of university students enrolled in mathematics classes suffer from high math anxiety. To the best of my ability, I can find neither claim in the two papers. Further, Dowker et. al. say that another survey, Ashcraft and Moore (2009) estimate that 17% of the general population suffer from math anxiety, but that paper (p. 199) explicitly disavows that reading. My most charitable reading is to assume that they have calculated these numbers from the data in the cited papers but, as they don't say, this is pure speculation on my part. Regardless, the papers do establish quite clearly that there is plenty of math anxiety to go around among university students.

[2] I have been unable to track down a firsthand source in English for this survey. When citing the same report from Science Daily, MacPherson (2016) also includes a reference to the Spanish language paper Pérez-Tyteca et. al. (2009) which seems to be the primary source for the claim. As I unfortunately do not speak Spanish, I haven't been able to verify the contents of the original paper.

What is Math Anxiety?

As with most complicated concepts there are plenty of definitions going around. One which is cited in many papers on the topic is that of Richardson and Suinn (1972) from the paper where they introduce the MARS-measurement for mathematics anxiety. They characterize it as:

> "a feeling of tension and anxiety that interferes with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations." (Richardson and Suinn, 1972, p. 551)

This definition focuses on the negative affective aspects of the phenomenon which express themselves, for example, through nervousness and discomfort. But, as Wigfield and Meece (1988, pp. 213-214) have shown, there is another dimension of math anxiety which instead expresses itself through negative attitudes towards the subject and a worry about performing poorly.

These two components are what Dowker et. al. (2016, p. 2) respectively call the *affective* and *cognitive* dimensions of math anxiety. Although the factors are clearly correlated with each other, Wigfield and Meece (1988, p. 214) have shown that they predict different effects in students. The affective dimension is a much stronger predictor of a student's low assessment of their own ability, performance, and expectancies. Further, only the affective dimension correlates with an actual poor performance and low interest in the subject. As it turns out, the cognitive dimension predicts a slight *increase* in both the value a student attaches to math and the effort they put in. As the authors interpret it, what this result means is that a little worry about doing well in class might motivate students. But if that worry becomes too strong, then the resulting affective response will overshadow any positive gains.

As more studies have been done on the connection between mathematical anxiety and performance, the negative correlations have kept showing up. In a meta-analysis encompassing 58 studies involving 6 137 university students in total, Hembree (1990, p. 38) found a negative correlation of 0.31 between grades and math anxiety ($p < 0.01$). Comparing with other studies in the meta-analysis, similar effects were noted when performance was measured with cognitive tests of computation, conceptualization, problem solving, abstract reasoning, and spatial ability.

One common outcome of anxiety disorders is that those who suffer from it try to avoid whatever events and experiences are associated with the feeling of anxiety (Abramowitz et. al. 2019, p. 46) and math anxiety is no different (Choe et. al. 2019, pp. 5-6). While not a great coping mechanism in everyday life, this is even worse as a learning strategy. As a result, students with math anxiety are more likely to avoid the studying required to learn the material.

What this depressing overview shows is that math anxiety is a serious problem for anyone who teaches a course with mathematical content. Now, formal logic requires pretty much the same skills from the student as any higher course in mathematics and, indeed, is taught by the mathematics departments at many universities. But as teachers of formal logic in a philosophy department we can expect our students to suffer from a higher level of math anxiety than their counterparts in the natural sciences. As such, its even more important to adopt strategies to try and mitigate the situation.

There is, however, some good news. A students' level of math anxiety isn't a fixed quantity, there are interventions which make a difference (Hembree, 1990, pp. 42-44). Better yet, the meta-analysis found that whenever an intervention decreases math anxiety, there is a corresponding improvement in performance. After successful intervention, students who previously exhibited high levels of math anxiety performed at a level approaching their low anxiety peers (Hembree, 1990, p. 43). Incidentally, this tells us that the phenomenon is one worth treating and that math anxiety isn't itself just a symptom of poor performance. It's one of its root causes.

However, not all interventions are created equal. Approaches which focus on classroom intervention through changes in the traditional curriculum or use of special equipment did not seem to have any effect. When looking at out-of-class interventions, the two types of treatment which proved most effective were versions of exposure therapy[3] and cognitive restructuring which focused on building self-confidence in students' ability to do mathematics (Hembree, 1990, p. 43). Exposure therapy is a form of psychological treatment for anxiety which focuses on experiencing anxiety inducing stimuli without engaging in avoidant behavior (Abramowitz et. al. 2019, pp. 11-13). Put simply, it's about re-training oneself to not produce a fear response to the stimuli. Cognitive restructuring is, instead, about exploring the way we think about the anxiety inducing stimulus in order to make it less harmful (Westbrook et. al. 2011, pp. 182-215). But, as Ma (1999, p. 532) comments in a later meta-analysis, there had at the time been little work done on interventions which exploit cognitive factors, such as skill-development and social communities of learning, which can be applied in the classroom.

Self-Efficacy and Mindset

Combining skill-development with the building of confidence brings us to the topic of *self-efficacy*. What this concept tries to capture is a students' belief that they can succeed in specific situations (Rozgonjuk et. al. 2020, p. 2). As applied to mathematics, we get the following definition from Ashcroft and Rudig (2012).

> "[S]elf-efficacy is an individual's confidence in his or her ability to perform mathematics and is thought to directly impact the choice to engage in, expend effort on, and persist in pursuing mathematics" (Ashcroft and Rudig, 2012, p. 249)"

Math self-efficacy, as it turns out, is highly negatively correlated with math anxiety (Cooper and Robinson, 1991; Akin and Kurbanoglu, 2011, p. 268; Rozgonjuk et. al. 2020, p. 6). One immediate question is, of course, which of these factors is the cause and which is the effect. Rozgonjuk et. al. (2020, p. 8) suggest, following Carey et. al (2016), a reciprocal model: that lower self-efficacy leads to higher anxiety which in turn leads to even lower self-efficacy. Turning this vicious cycle on its head, however, suggests a potential solution. Intervening to increase student's self-efficacy can help mitigate anxiety.

One direct way which teachers can improve self-efficacy is by helping their students experience success which they attribute to either their own ability or efforts (Ambrose et. al. 2010, pp. 77-79). Essentially, students who establish a track record of succeeding at their assignments begin to expect that they can do so again. However, it's important to avoid the impression that the

[3] Hembree (1990, p. 43) specifically mentions systematic desensitization as the most common behavioral treatment, which is a form of exposure therapy (Abramowitz et. al. 2019, p. 14).

students' success was caused by factors outside of their control, such as an assignment that was too easy, as that wouldn't support the belief that their success can be repeated.

As Dowker et. al. (2016, p. 3) note, self-efficacy shouldn't be reduced to a students' self-assessment of their current performance; it also includes their beliefs about their ability to improve in mathematics and take control of their learning. This points towards another way teachers can encourage math self-efficacy in students: namely, through promoting a dynamic view of intelligence.

A dynamic view of intelligence, a growth mindset, is a view that sees intellectual ability as something which can be developed over time through training (Yeager and Dweck, 2012, pp. 303-304). By contrast, students who have a fixed view of intelligence tend to think that their abilities are set in stone. As such, they believe that the effort they put into learning matters less than how naturally gifted they are at the subject in question.

It should come as little surprise that there is a connection between a students' self-efficacy and mindset. After all, students who believe that improving their abilities is within their power have an easier time believing that they can face future challenges. As Dweck and Master (2009, p. 131) report, when comparing the development of students' self-efficacy while taking a course, those who hold a dynamic view of intelligence show an increase while their fixed mindset counterparts show a decrease.

Even better for those of us who hope to tackle math anxiety through developing self-efficacy, students can be taught to adopt growth mindsets (Dweck and Master, 2009, pp. 136-137; Ambrose et. al., 2010, pp. 201-202). Although these studies haven't dealt with specifically math related self-efficacy, there are some indications that the same relationship holds. In a pilot study trialing the effects of an intervention designed to promote a growth mindset to undergraduate students of statistics, Samuel and Warner (2021, p. 217) found that the students reported both an increase in the relevant self-efficacy and a decrease in math anxiety.

In this way, self-efficacy provides a way for us to tackle math anxiety as teachers rather than therapists. Unfortunately, this also means that we carry the risk of causing great harm. In a series of studies, Rattan et. al. (2011, pp. 735-736) explored the effects of teacher communication and found that feedback which conveyed a fixed view of mathematical ability led to students reporting both lower motivation and expected performance. As such, the very least we must do is ensure that we communicate the idea that mathematical ability comes from practice and not some sort of innate ability.

Co-operation and Course Climate

Having discussed these cognitive factors, I want to turn next to the part played by the social and environmental circumstances of the course. One such dimension which MacPherson (2016, pp. 130-131) emphasizes is the role of empathy in the classroom. By this he means both *cognitive empathy*, understanding math anxiety from the students perspective, and *affective empathy*, which encompasses that the teacher shows that they care about their students struggle with anxiety. The former, as he points out, is especially important for teachers of logic who haven't themselves struggled with math anxiety.

Displays of affective empathy also help in fostering a course climate where the students feel supported which, in turn, helps build motivation (Ambrose et. al. 2010, pp. 79-82). Now, although the path analysis model performed by Lin et. al. (2017, pp. 341-342) didn't find a direct connection between a supportive classroom climate and reduced math anxiety, it did show that the increase in motivation connected to a supportive climate leads to a corresponding reduction in anxiety.

Another way we leverage social factors to alleviate anxiety is through the promoting of group work. As MacPherson (2016, pp. 132-134) reports, numerous studies have found that students who are taught via methods which focus on co-operative learning experience less math anxiety than their peers who are taught via traditional methods.

This coheres well with the finding by Cooper et. al. (2018, pp. 10-13) that when working in a group where they're comfortable students generally experience less anxiety as a result of realizing that others also struggle with the concepts of the course. In fact, getting help from a friend was the only teaching and learning method where Bjälkebring (2019) found that students with high math anxiety are more likely to report using and being helped by. As such, by encouraging group work we can make this strategy more easily available.

Putting It All Into Practice

Now, with this overview in hand its finally time to turn to concrete strategies.

One of the very first things to do when starting the course is simply to address the issue directly: talking to the students about the fact that some of them likely experience math anxiety and that while that is an obstacle it's one which can be overcome and that the course has been designed with this in mind. We should also tell them that research has shown that many students who suffer from anxiety and believe that they lack mathematical ability have managed to do well through sustained engagement with the material.

This helps demonstrate empathy towards the students who do suffer from it, both in the cognitive sense that their experience is taken seriously and in the affective sense that their teacher cares about helping them deal with the problem. It's also a great opportunity to briefly discuss the difference between fixed and growth mindsets and how the latter is both better supported by the evidence and has been shown to help alleviate anxiety. This kind of intervention is what produced the positive results both in the study performed by Samuel and Warner (2021) and the one reported on by Ambrose et. al (2010, pp. 201-202).

The second major strategy is to assign the students regular homework which contributes to their final grade. By doing so we can provide the students with an early opportunity for success, especially if we make the first assignment somewhat easy, to help build self-efficacy (Ambrose et. al. 2010, pp. 86-87). Further, regular homework provides the opportunity for students to accumulate practice with the material throughout the course, while the fact that it contributes to their grade provides additional incentive to overcome any avoidant behavior caused by anxiety. Although far from the sophisticated treatment a therapist can provide, this also provides the student with exposure to the anxiety inducing situation which, if they get sufficient support to handle it, may help desensitize them from the situation.

These homework assignments also provide frequent opportunities for formative feedback so that students can both evaluate their progress and improve the understanding gained through practice (Ambrose et. al. 2010, pp. 137-141). By including praise of effort, engagement, and improvement in the feedback process we can also help encourage the students towards adopting a growth mindset (Dweck, 2007, pp. 36-37). If we really want to harness this process, we can allow the students the opportunity to revise their homework in light of the feedback before the assignment is graded. By encouraging them to, and perhaps giving credit for, specifying how they've incorporated the feedback they received, we help ease them into the practice of viewing their ability dynamically.

The final proposal I want to consider is how we can encourage co-operative learning in the course. As MacPherson (2016, p. 134) suggests this can be done in the classroom by grouping students together to work on proofs and problems instead of having the teacher write them out on the board. By weaving these peer discussions into the lectures, we can both directly alleviate math anxiety in the situation and put the *process* of logical problem-solving front and center. Instead of the teacher presenting the solutions from out of nowhere, the students get to both engage with and observe the messy and mistake-laden process of getting there.

We can also assign the students to study groups outside of the classroom, with the stated goal of co-operatively working on assignments. This provides the opportunity for peer support for learning as well as reducing anxiety by showing the students that the difficulties they face in the course are shared. One advantage of the teacher grouping students together is that it lowers the risk that some socially isolated pupil gets left behind by established social groups (Bjälkebring, 2019, p. 4). On the other hand, as Cooper et. al. (2018, pp. 10-11) emphasize, anxiety is only reduced if the student feels comfortable with their working partner; it increases if they do not. As such, grouping students together brings with it additional risk, although that might be mitigated by informing the students that they can be transferred to another study group if they for whatever reason feel incompatible with the one they've been assigned.

Conclusion

Math anxiety is a serious obstacle for teaching formal logic to philosophy students as it is a common condition which negatively affects their ability to learn the subject. In searching for methods of mitigation I have in this essay explored some of the connections between the phenomenon of math anxiety and the social and cognitive factors which affect it. Self-efficacy, which can be promoted through successful performance and the adoption of a growth mindset, turns out to play a key role for helping students overcome anxiety. Further, working co-operatively with peers seems to lessen anxiety apart from its other benefits for learning. These insights can help us design a course which is structured so as to minimize the effects of math anxiety through the use of the following strategies:

(1) Engaging with the students about the problem in order to build an empathetic and supportive environment for the course.
(2) Promote a growth mindset both through informing the students about what it is and its benefits and by centering feedback on effort and process.
(3) Assign regular homework with the opportunity to respond to formative feedback.
(4) Encourage the students to work co-operatively in groups both inside and outside the classroom.

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